

slope S_c is equal to 1/8 of the Darcy-Weisbach friction factor **f**. Table 1 lists the twelve types of profiles (**Ponce, 2014**).

Table 1. The twelve (12) types of water-surface profiles.													
Family		Character	Rule	S _o > S _c	$S_o = S_c$	S _o < S _c	<i>S</i> ₀ = 0	<i>S</i> ₀ < 0					
I		Retarded (Backwater)	$1 > F^2 < (S_o / S_c)$	S ₁	C ₁	M ₁	-	-					
II	A	Accelerated (Drawdown)	$1 < F^2 < (S_o / S_c)$	S ₂	-	-	-	-					
	в	Accelerated (Drawdown)	$1 > F^2 > (S_o / S_c)$	-	-	M ₂	H ₂	A ₂					
Ш		Retarded (Backwater)	$1 < F^2 > (S_o / S_c)$	S ₃	C ₃	M ₃	H ₃	A ₃					

In this article, we describe the M_1 water-surface profile, a retarded (backwater) subcritical/subnormal profile, which may well be the most common profile in practice. The M_1 profile depicts flow in a mild channel, upstream of a reservoir (Fig. 1). We present two examples and show the respective online calculations.

 M_1 PROFILE: $y > y_n > y_c$

 $S_y = 0$ (=> normal depth) $S_y = S_o$ (=> horizontal)



Fig. 1 M₁ water-surface profile.

2. GOVERNING EQUATION

Chow (1959) has presented the classical way of expressing the governing equation of steady, gradually varied flow. A more cogent, dimensionless presentation, focusing on critical slope, has been advanced by **Ponce (2014)** and is presented here.

In terms of critical slope, the general equation for flow-depth gradient dy/dx is:

$$\frac{dy}{dx} = \frac{S_o - (P/T) (T_c/P_c) S_c F^2}{1 - F^2}$$
(1)

in which S_o = channel (bottom) slope, S_c = critical slope, P = wetted perimeter, T = channel top width, T_c = channel top width at critical flow, P_c = wetted perimeter at critical flow, and F = Froude number. The Froude number is defined as follows: $F = v/(gD)^{1/2}$, in which v = mean flow velocity, g = gravitational acceleration, and D = hydraulic depth, in which D = A/T.

For $(P/T) = (P_c/T_c)$, Eq. 1 reduces to:

$$\frac{dy}{dx} = \frac{S_o - S_c F^2}{1 - F^2}$$
(2)

For conciseness, the flow-depth gradient may be written as:

$$S_{y} = \frac{dy}{dx}$$
(3)

Substituting Eq. 3 into Eq. 2, the flow-depth gradient is:

$$\frac{S_y}{S_c} = \frac{(S_o / S_c) - F^2}{1 - F^2}$$
(4)

Equation 2, or Equation 4, its reduced form, is the steady gradually varied flow equation (Fig. 2). The depth gradient S_y is shown to be a function *only* of the following variables: (1) channel (bottom) slope S_o , (2) critical slope S_c , and (3) Froude number **F**.



Fig. 2 Definition sketch for energy balance in open-channel flow.

3. ONLINE CALCULATION: EXAMPLE A

We pose an example of the calculation of an M₁ profile in a natural channel using the online calculator **ONLINE_WSPROFILES_21**. The following box shows the input data.

Example A: Input Data

- Discharge Q = 100 m³/s
- Bottom width = 20 m
- Side slope z = 2 (z H : 1 V)
- Bottom slope $S_o = 0.0005$
- Manning's n = 0.03
- Normal depth at the downstream boundary $y_d = 5 \text{ m}$
- Number of computation intervals: n = 100
- Number of tabular output intervals: *m* = 20

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M1, PROFILE: $y > y_c > y_c$ Description Sy=0 (=> normal depth) Sy=S_c (=> horizontal) Example Using hereich Sy=0 (=> normal depth) Sy=S_c (=> horizontal) Y>y_c subcritical Y>y_c subcritical Description Example Example Building of the second Sy=Computational IPUT DATA: Sy=Computational Select: Using function Using function Enter discharge Q (m ² h) (thi): The bottom width B (m) (thi: 20 = Enter side slope z (z H:1 V): Enter discharge Q (m ² h) (thi): There bottom width B (m) (thi: 20 = Enter side slope z (z H:1 V): 2 Enter bottom slope So (mvm) (thi): There bottom width B (m) (thi: 20 = Enter side slope z (z H:1 V): 2 Enter bottom slope So (mvm) (thi): There bottom width B (m) (thi: 20 = Enter sloe slope z (z H:1 V): 2 Enter number of computational intervals n (suggested range So-200) [If left blank, a default value of 100 will be used]: 20 20 Enter number of actual on and many sin Side slope z = 2 m/m Maring's n = 0.03 Bottom width B = 20 m Side slope z = 2 m/m Maring's n = 0.03 Bottom slope So m Side slope z = 2 m/m														
$\begin{tabular}{ c c c c } \hline W, PROFILE: $y > y_{1} > y_{1} \\ $y_{1} > y_{2} > y_{2} \\ $y_{1} > y_{2} > y_{2} \\ $y_{2} > y_$	Computational depth interval $\Delta y = 0.0204 \text{ m}$ Tabular output depth interval $(\Delta y)_t = 0.102 \text{ m}$													
$ \begin{array}{c c} & & & \\ $	Normal depth y _n = 2.962 m Normal-depth Froude number F _n = 0.268													
Image: Sign of	Depth (m)	Area (m ²)	Velocity (m s ⁻¹)	Velocity head (m)	Specific head (m)	Wetted perimeter (m)	Hydraulic radius (m)	Friction slope (m/m)	Average slope (m/m)	Specific head difference (m)	Length increment (m)	Total lengti (m)		
y > y, subcritical y, > y, mid y > y, subcritical y, > y, mid Destative 10 ct 10 ct 10 ct 10 ct <t< td=""><td>5</td><td>150</td><td>0.67</td><td>0.023</td><td>5.023</td><td>42.36</td><td>3.54</td><td>0.00007411</td><td>· ·</td><td>-</td><td>•</td><td>0</td></t<>	5	150	0.67	0.023	5.023	42.36	3.54	0.00007411	· ·	-	•	0		
Sigest	4.898	145.94	0.69	0.024	4.922	41.9	3.48	0.00008004	0.00007942	0.02	47.8	237.		
INPUT DATA: 25 Select: U.S. Constants within the constructional intervals in (suggested range 50-200) [II left blank, a default value of 100 will be used]; 20 Select: U.S. Constants within the downstream boundary y ₄ (m) [II] (a subcritical subcritic	4.796	141.93	0.7	0.025	4.821	41.45	3.42	0.00008656	0.00008589	0.02	48.5	479.		
NPUT DATA: 20 4 25 30 2 35 4 45 45 45 50 50 50 50 50 50 50 50 50 50 50 50 50 50 <	4.694	137.96	0.72	0.027	4.721	40.99	3.37	0.00009376	0.00009301	0.02	49.3	724		
INPUT DATA: 25	4.592	134.03	0.75	0.028	4.621	40.54	3.31	0.00010172	0.00010089	0.02	50.2	973		
B units (methic) 30 4 Select: U.S. Customy units 6 6 Ender discharge Q of uppt I intervals m (suggested range 50-200) [I left blank, a default value of 10 will be used]: 20 45 Enter Dottom Bop Ba, (min) [1/1]: 0.0005 Enter Manning's n: 0.003 6 Enter Dottom slope Ba, (min) [1/1]: 0.0005 Enter Manning's n: 0.003 55 0 Enter Dottom slope Ba, (min) [1/1]: 0.0005 Enter Manning's n: 0.003 55 0 55 Enter Number of computational intervals n (suggested range 50-200) [I left blank, a default value of 10 will be used]: 000 70 3 Enter Number of tabular output intervals m (suggested range 10-50) [I left blank, a default value of 10 will be used]: 20 75 3 ECHO OF INPUT: Discharge Q = 100 m ³ s ¹ Bottom width B = 20 m Side slop z = 2 m/m 30 36 30 30 Manning's n = 0.03 Bottom slope Sa = 0.0005 m/m 56 56 56 56 56	4.49	130.14	0.77	0.03	4.521	40.08	3.25	0.00011053	0.00010961	0.02	51.3	1227		
Select: U.S. Customary units 65 46 Enter discharge 0. (m ⁴ /b) (chs) 1: 100 Enter bottom width 8 (m) (th; 30 Enter side slope z (z H:1 V): 2 Enter bottom slope S0, (m ⁴ /b) (chs) 1: 100 Enter bottom width 8 (m) (th; 30 Enter side slope z (z H:1 V): 2 Enter bottom slope S0, (m ⁴ /b) (H1) 0.00005 Enter Manning's n: 0.03 50 3 Enter bottom slope S0, (m ⁴ /b) (H1) 0.0005 Enter Manning's n: 0.03 50 3 Enter bottom slope S0, (m ⁴ /b) (H1) 0.0005 Enter Manning's n: 0.03 50 3 Enter bottom slope S0, (m ⁴ /b) (H1) 0.0005 Enter Manning's n: 0.03 65 3 Enter number of computational intervals n (suggested range 10-50) (H1 lett blank, a default value of 100 will be used 1: 100 70 3 Enter NUPT: Enter NUPT: 50 3 50 3 3 3 Discharge Q = 100 m ⁴ s ⁻¹ Bottom slope S ₂ = 0.0005 m/m Side slope z = 2 m/m 90 3 3 3 3 3 3 3 3 3 3 3 3 3 3	4.389	126.29	0.79	0.032	4.421	39.63	3.19	0.00012031	0.00011929	0.02	52.5	148		
Enter discharge Q (m ² /s) (cis) : 100 Enter bottom width B (m) [th; 20 Enter raide slope z (z H:1 V); 2 40 4 Enter discharge Q (m ² /s) (cis) : 100 Enter Manning's n: 0.03 50 5 5 5 5 5 5 5 5 60 3 60	4.287	122.49	0.82	0.034	4.321	39.17	3.13	0.00013119	0.00013006	0.02	54	175		
AB 48 66 8 66 8 7	4.185	118.72	0.84	0.036	4.221	38.71	3.07	0.00014333	0.00014206	0.02	55.7	202		
Enter Mannings n: 0.03 001 </td <td>4.083</td> <td>115</td> <td>0.87</td> <td>0.039</td> <td>4.121</td> <td>38.26</td> <td>3.01</td> <td>0.00015689</td> <td>0.00015547</td> <td>0.02</td> <td>57.7</td> <td>231</td>	4.083	115	0.87	0.039	4.121	38.26	3.01	0.00015689	0.00015547	0.02	57.7	231		
Enter Elw depth at the downstream boundary y _d (m) (ft) (a subcritical subnormal flow depth) [if left blank, program will use 1.01*normal depth]: 50 <	3.981	111.32	0.9	0.041	4.022	37.8	2.94	0.00017209	0.00017050	0.02	60.2	260		
5 000000000000000000000000000000000000	3.8/9	107.68	0.93	0.044	3.923	37.35	2.88	0.00018918	0.00018739	0.02	63.3	29		
Enter number of computational intervals in (suggested range 50-200) [II left blank, a default value of 100 will be used]: 100 170 2 Enter number of tabular output intervals in (suggested range 10-50) [II left blank, a default value of 10 will be used]: 20 75 30 75 ECHO OF INPUT: Discharge Q = 100 m ² s ⁻¹ Bottom width B = 20 m Side slope z = 2 m/m 80 90 30 Maning's n = 0.03 Bottom slope S ₀ = 0.0005 m/m 56 3 30 30	3.111	104.08	0.90	0.047	3.024	30.09	2.02	0.00020843	0.00020041	0.02	70.0	324		
Enter number of tabular output intervals m (suggested range 10-50) [11 left blank, a default value of 10 will be used]: 20 70 5 ECHO OF INPUT: 85 85 86 85 86	3.673	97.01	1.03	0.054	3.628	35.98	2.70	0.00025020	0.00022791	0.02	72.3	300		
ECHO OF INPUT: 88 Discharge Q = 100 m ³ s ⁻¹ Bottom width B = 20 m Side slope z = 2 m/m 90 Marning's n = 0.03 Bottom slope S ₀ = 0.0005 m/m	3.471	93.53	1.07	0.058	3.53	35.52	2.63	0.00028298	0.00028002	0.02	88.8	440		
EcHO O FINPUT: 85 5 Discharge Q = 100 m ² s ⁻¹ Bottom width B = 20 m Side slope z = 2 m/m 90 5 Manning's n = 0.03 Bottom slope S ₂ = 0.0005 m/m 65 3	3.37	90.1	1.11	0.063	3.432	35.07	2.57	0.00031507	0.00031169	0.019	103.2	488		
Discharge Q = 100 m ³ s ⁻¹ Bottom width B = 20 m Side slope z = 2 m/m 95 3 Manning's n = 0.03 Bottom slope S ₀ = 0.0005 m/m 95 3	3.268	86.71	1.15	0.068	3.335	34.61	2.51	0.00035186	0.00034797	0.019	127.2	547		
Manning's n = 0.03 Bottom slope S ₀ = 0.0005 m/m	3.166	83.36	1.2	0.073	3.239	34.16	2.44	0.00039420	0.00038972	0.019	174.3	623		
	3.064	80.05	1.25	0.08	3.143	33.7	2.38	0.00044315	0.00043796	0.019	307.7	743		
Specified flow denth at the downstream boundary, y ₄ = 5 m	2.962	76.79	1.3	0.086	3.048	33.25	2.31	0.00050000	0.00049396	0.019	3136.7	1311		
						,						_		

Fig. 3 (a) Example A: Input.



Output. The results show that the (flow) depth at the downstream boundary ($y_d = 5$ m) will decrease gradually to the normal depth in the upstream end $y_n = 2.962$ m. The total distance, from downstream boundary to upstream end, is: L = 13,114.5 m.

4. ONLINE CALCULATION: EXAMPLE B

We pose an example of the calculation of an M₁ profile in a lined channel using the online calculator **ONLINE_WSPROFILES_21**. The following box shows the input data.

Example B: Input Data

- Discharge $Q = 50 \text{ m}^3/\text{s}$
- Bottom width = 6 m
- Side slope z = 1 (z H : 1 V)
- Bottom slope $S_o = 0.002$
- Manning's n = 0.015
- Normal depth at the downstream boundary $y_d = 5 \text{ m}$
- Number of computation intervals: n = 100
- Number of tabular output intervals: m = 10

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onnine_wspronies_41: M1 water-surface prome															
M_1 PROFILE: $y > y_c > y_c$	Description					[0	Click	on to	op of	figure	e to ex	pand]			
$S_i = 0$ (∞) normal depth) $S_i = S_i$ (∞) horizontal) $\frac{\mathbf{v}_i}{\mathbf{y}_i}$ $\mathbf{y} > \mathbf{y}_i$ subnormal	Example Summary Reference	Com	nputational mal depth y	depth inte n = 1.82	rrval∆y = 1 m	0.0318 n Nor	n mal-depth	Tabula Froude nu	r output dep imber F _n =	oth interval (0.922	∆y) _t = 0.318	m			
$y > y_c$ subcritical $S_u < S_c$ $y_c > y_c$ mild	Disclaimer		k	Depth (m)	Area (m²)	Velocity (m s ⁻¹)	Velocity head (m)	Specific head (m)	Wetted perimeter (m)	Hydraulic radius (m)	Friction slope (m/m)	Average slope (m/m)	Specific head difference (m)	Length increment (m)	Total length (m)
			0	5	55	0.91	0.042	5.042	20.14	2.73	0.00004872	•	·	•	0
INPUT DATA:			10	4.682	50.02	1	0.051	4.733	19.24	2.6	0.00006292	0.00006211	0.031	15.9	158.9
St units (metric)		20	4.364	45.23	1.11	0.062	4.427	18.34	2.47	0.00008253	0.00008140	0.031	15.9	317.9	
Select: U.S. Customary units [Choose S.I. Units or U.S. Customary units]		30	4.046	40.65	1.23	0.077	4.123	17.44	2.33	0.00011018	0.00010856	0.03	15.9	477.1	
Enter discharge Q (m ³ /s) [cfs] : 50 Enter bottom width B (m) [ft]: 6 Enter side slope z (z H:1 V): 1	j		40	3.729	36.27	1.38	0.097	3.825	16.55	2.19	0.00015012	0.00014776	0.03	15.9	636.4
Enter bottom slope S ₂ (m/m) [ft/ft]: 0.002 Enter Manning's n: 0.015		50	3.411	32.1	1.55	0.124	3.534	15.65	2.05	0.00020949	0.00020594	0.029	16	796.1	
			70	2 775	20.12	2.05	0.101	2.09	13.85	1.91	0.00030079	0.00023524	0.027	16.3	1118.4
Enter now depth at the downstream boundary yd (m) (it) (a subcritical subnormal now depth) (if left blank, program will use 1.01 normal d	Jeptn J: 6		80	2.457	20.78	2.41	0.295	2,752	12.95	1.6	0.00069347	0.00067784	0.022	16.8	1283.4
Enter number of computational intervals n (suggested range 50-200) [If left blank, a default value of 100 will be used]: 100			90	2.139	17,41	2.87	0.42	2.559	12.05	1.44	0.00113600	0.00110709	0.016	18,4	1457.8
Enter number of tabular output intervals m (suggested range 10-50) [If left blank, a default value of 10 will be used]: 10			100	1.821	14.24	3.51	0.628	2.449	11.15	1.28	0.00200000	0.00194131	0.006	94.2	1771.6
ECHO OF INPUT:									_		_				
Discharge Q = 50 m ³ s ⁻¹ Bottom width B = 6 m Side slope z = 1 m/m						Fia.	4 (b) E	Exar	nple	e B: (Outp	ut.		
Manning's n = 0.015 Bottom slope S ₀ = 0.002 m/m						.9.	. (- / -							
Specified flow depth at the downstream boundary yd = 5 m															
Number of computational intervals n = 100 Number of tabular output intervals m = 10															



Output. The results show that the (flow) depth at the downstream boundary ($y_d = 5$ m) will decrease gradually to the normal depth in the upstream end $y_n = 1.821$ m. The total distance, from downstream boundary to upstream end, is: L = 1,771.6 m.

5. SUMMARY

An online calculation of an M_1 open-channel flow water-surface profile is shown in detail. Two examples using the script **ONLINE_WSPROFILES_21** demonstrate the utility of using this online digital tool for the accurate and expedient calculation of an M_1 water-surface profile.

REFERENCES

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